It is found from this figure that the gain increases rapidly with the increasing height for any radio frequency and becomes infinite at the corresponding cutoff heights where the radio frequency is equal to the local gyrofrequency, and therefore the phase velocity of the wave is zero. The radio frequency amplified for a constant electron stream velocity is plotted vs the height from the earth surface in Fig. 2. Fig. 3 shows the calculated gain vs the height for a constant electron stream velocity. In this figure the dashed curves indicate equifrequency contours. As a numerical example, the gain of about 170 db per hundred kilometers is obtained for $f=3$ kc, $c/U=60$, at the height of $h/r_0=4.7$. Note that the assumed condition, $|\delta| \ll n_2$ is still valid for the case of such large amplification. Extremely large gain is calculated for relatively high frequencies at $c/U=100$ as shown in Fig. 3, but it should be noted that in this region the radio frequencies are very near the local gyrofrequencies or the cutoff frequencies, for example, the gyrofrequency at $h/r_0=3.0$ is about 20.0 kc as seen in Fig. 1.

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Comments on "Closed Form Analysis for the Radiation Pattern of the Modulated Antenna"*

In Section VI of the paper by Ishimaru and Bernard,[1] the sign of the phase variation of the modulated aerial for minimum sidelobe level has been determined. However, the reasons given for the required result, *i.e.*, $\beta'<0$ [(25) of the paper], appear to be in error.

For a phase modulated wave, the radiation pattern of the aerial is given by:

$$g(u') = \frac{1}{2}\int_{-1}^{1} e^{j[u'x-\beta \sin m\pi x]}dx \qquad (1)$$

where

$$u' = u - u_s$$
$$= \frac{2\pi a}{\lambda}(\sin\theta - c/v_s). \qquad (2)$$

The symbols, except for u' which has been used to prevent confusion, have the same meaning as given in the paper by Ishimaru and Bernard.

The authors express the above equation in terms of an array factor, and the Anger function, $J_\nu(\beta)$:

$$g(u') = \frac{\sin \nu m\pi}{m \sin \nu\pi} J_\nu(\beta') \qquad (3)$$

where

$$J_\nu(\beta') = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(\nu\varphi-\beta'\sin\phi)}d\phi$$
$$\nu = u'/m\pi$$
$$\beta' = (-1)^{m-1}\beta.$$

For the choice of a suitable value for v_s/c (the average relative phase velocity along the array), it is first necessary to consider the array factor:

$$\frac{\sin \nu m\pi}{\sin \nu\pi}.$$

The maximum value of this function occurs for values of $\nu=0$, ± 1, ± 2, etc. To determine the appropriate value of ν for an aerial which has its maximum radiation in the endfire direction, *i.e.*, $\theta=+90°$, consider (2):

$$\nu = u'/m\pi = \frac{2a}{m\lambda}(1-c/v_s). \qquad (4)$$

If $\nu \simeq 0$, then the average phase velocity along the array must approach that of free space. This does not yield a practical solution for a phase modulated aerial. Consequently if the supergain ratio is to be kept to a minimum, and the phase velocity less than that of free space, then from (4), $\nu=-1$. The fact that ν must be negative has been overlooked in Section VI of the paper.

To choose the correct sign of β' for an array with minimum sidelobe level, it is necessary to consider the Anger Function, $J_\nu(\beta')$, which is plotted as a function of $\pm\nu$ and $\mp\beta'$ in Fig. 2 of the paper.[1] The region of minimum sidelobe level is seen to occur for values of positive ν and β'. Since $\nu \leq -1$ for the endfire array under consideration, β' must also be negative, because:

$$J_{-\nu}(-\beta') \equiv J_{+\nu}(+\beta').$$

Thus, although the authors have given the correct sign for β', *i.e.*, $\beta'<0$, ν must always be negative and not positive for the endfire array under consideration.

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* Received July 9, 1962.

[1] A. Ishimaru and G. D. Bernard, "Closed form analysis for the radiation pattern of the modulated antenna," IRE Trans. on Antennas and Propagation, vol. AP-10, pp. 151–159; March, 1962.

Author's Reply[2]

The authors wish to thank Mr. Thomas for pointing out a confusing section of our paper. However, the reason given for (25) is not in error, and the fact that ν must be negative in the visible region of the slow wave antenna has not been overlooked, but was explicitly stated immediately preceding (23).

Our argument for requiring $\beta'<0$ may be more clearly restated in the following manner:

Please refer to Figs. 3 and 4 of our paper, which are graphs of $g(u')$ vs $-u'/\pi$ $(u'/\pi = m\nu)$ for β' *positive*. Since $u-u_s<0$ in the visible region, the function $g(u')$ to the "left side" of the origin $(m\nu<0)$ is available for use as the visible region. However, the lowest sidelobes occur on the "right side" $(m\nu>0)$ of the origin. But this "side" $(m\nu>0)$ is *also* available for use as the visible region since

$$J_{-\nu}(\beta') \equiv J_\nu(-\beta')$$

which means we must make $\beta'<0$, in which case the visible region will fall in the region of lowest sidelobes for the endfire antenna.

We would also like to take this opportunity to mention that the method presented has application to broadside aperture antennas in which case $u=ka \sin\theta$.

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[2] Received July 27, 1962.

The Radiation Pattern of Long Thin Antennas for Short-Pulse Excitation*

There has been considerable recent interest both in the transient response of antennas,[1] and in the scattering of short-pulses by conducting obstacles.[2] This has been at least partially motivated by increased use and interest in short-pulse wideband radars. The analysis of short-pulse radiation and scattering patterns is also of general theoretical interest, in that the results may be more amenable to simple physical interpretation than are the corresponding CW results.

The purpose of this note is to investigate the radiation from a traveling-wave pulse on a long thin antenna. It is shown that the radiation results solely from the discontinuity at the two ends of the antenna, and is in the form of two pulses each of which has a single broad-lobe angular dependence. The familiar multilobed traveling-wave radiation pattern results from the overlap (interference) of these two pulses. The condition for the resolvability of the two end contributions is discussed, and the possible application of the results to the further study of traveling-wave antennas and scattering problems is noted.

A long thin antenna supporting a traveling wave of current of the form $I_0 \exp j\omega(t-z/v)$ has a far-field radiation pattern given by[3]

$$H_\phi = \frac{I_0}{4\pi r}\frac{\sin\theta}{\cos\theta - 1/p}(e^{j\omega u_2} - e^{j\omega u_1}), \qquad (1)$$

where

$$p = v/c, \qquad u_1 = t - \frac{r_1}{c},$$
$$u_2 = t - \frac{r_2}{c} - \frac{b}{v} \text{ (see Fig. 1)}$$

and v is the phase velocity of the traveling wave on the antenna.

* Received July 16, 1962.

[1] R. W. P. King and H. J. Schmitt, "The transient response of linear antennas and loops," IRE Trans. on Antennas and Propagation, vol. AP-10, pp. 222–228; May, 1962.

[2] L. Peters, Jr., "End-fire echo area of long thin bodies," IRE Trans. on Antennas and Propagation, vol. AP-6, pp. 133–139; January, 1958.

[3] J. D. Kraus, "Antennas," McGraw-Hill Book Co., Inc., New York, N. Y., p. 151; 1950.

To investigate the response of a long thin body to pulse excitation, it is assumed that the traveling wave is of the form[4]

$$I = \begin{cases} 0 & t < z/v \\ I_0 \sin \omega_0 (t - z/v) & \dfrac{z}{v} < t < \dfrac{z}{v} + T. \\ 0 & t > \dfrac{z}{v} + T \end{cases} \quad (2)$$

The spectrum of the traveling waves composing I is obtained from

$$I(\omega) = \frac{I_0}{\sqrt{2\pi}} \int_{z/v}^{z/v+T} \sin \omega_0 (t - z/v) \cdot \exp\left[-j\omega(t - z/v)\right] dt. \quad (3)$$

The response of the antenna to pulse excitation is then given by

$$H_\phi = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{1}{4\pi r} \frac{\sin \theta}{\cos \theta - \dfrac{1}{p}} \cdot (e^{j\omega u_2} - e^{j\omega u_1}) I(\omega) d\omega \quad (4)$$

which, from the definition of $I(\omega)$, is

$$H_\phi = \frac{I_0}{4\pi r} \frac{\sin \theta}{\cos \theta - \dfrac{1}{p}} \cdot \left[a\left(t - \frac{r_2}{c} - \frac{b}{v}\right) - a\left(t - \frac{r_1}{c}\right) \right] \quad (5)$$

where

$$a(t) = \begin{cases} 0 & t < 0 \\ \sin \omega_0 t & 0 < t < T. \\ 0 & t > T \end{cases} \quad (6)$$

Thus, the radiation pattern is composed of contributions from the two ends of the antenna, each of which results in an angular dependence given by

$$X_p(\theta) = \frac{\sin \theta}{\dfrac{1}{p} - \cos \theta} \cdot \quad (7)$$

This angular pattern is shown for several values of p in Fig. 2. There is a maximum for cos $\theta = p$, so that the maximum shifts towards the antenna axis as the phase velocity increases, similar to an end-fire array.

For the steady-state case, the returns from the two ends overlap, and the angular dependence is

$$X_{ss}(\theta) = \frac{\sin \theta}{\dfrac{1}{p} - \cos \theta} \cdot \left\{ 2 \sin \left[\frac{\omega b}{2c} \left(\frac{1}{p} - \cos \theta \right) \right] \right\}. \quad (8)$$

The term within the curly brackets is due to the interference of the contributions from the two ends, and gives rise to the familiar multilobed pattern shown in Fig. 3, having an envelope given by the corresponding curve in Fig. 2.

The pulse pattern, $X_p(\theta)$, is obtained only if the returns from the two ends are nonoverlapping in time; *i.e.*, only if

$$\frac{r_2}{c} + \frac{b}{v} > T + \frac{r_1}{c},$$

[4] It is assumed that the antenna is essentially nondispersive so that phase and group velocities are equal to one another and constant over the bandwidth of interest.

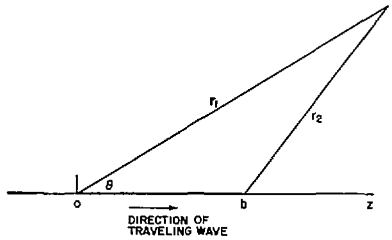

Fig. 1—Antenna geometry.

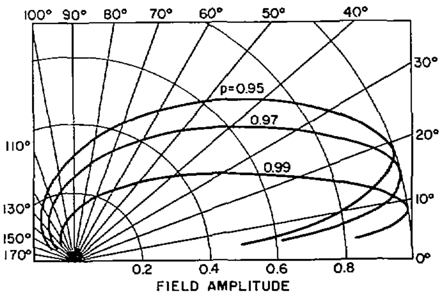

Fig. 2—Normalized pulse radiation pattern.

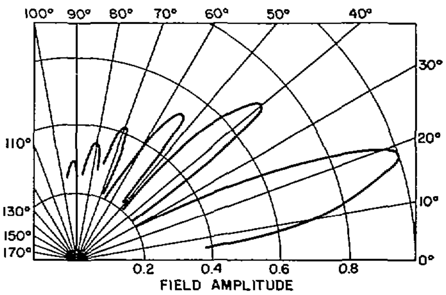

Fig. 3—Normalized steady-state radiation pattern. $b = 5\lambda$, $p = 0.95$.

from which it follows that

$$b > \frac{cT}{\dfrac{1}{p} - \cos \theta} \cdot \quad (9)$$

For example, for $p = 0.95$, cos $\theta = 0.95$, and $T = 10^{-9}$ sec, we require $b > 3$ meters.

Reflections from the ends of the antenna and the dispersive nature of the propagation along the antenna would, of course, modify these results. An experimental investigation of the transient response of traveling-wave antennas should therefore provide information on both reflections and dispersion in such antennas.

The above results should also be applicable to the scattering of short-pulses by long, thin obstacles. Peters[2] has essentially used $X_{ss}^4(\theta)$ as the angular dependence of the back-scattering cross section of such obstacles. For short-pulse scattering, the returns from the two ends may be resolved in time, and the angular dependence of the scattering would then be determined by $X_p^4(\theta)$.

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Tabulation of Calculated Equatorial Plane Radiation Fields Produced by a Circumferential Slot on a Cylinder*

Tabulations of the computations for the equatorial plane radiation fields produced by a sinusoidal variation of axial electric field along a half wavelength circumferential slot on a metal cylinder of radius a are given for sizes of cylinder $1 \leq ka \leq 30$.

Although the radiation patterns corresponding to most of the computations have been reported,[1] it seems worthwhile to give the actual computations here in tabular form. They were performed on an IBM-704 electronic computer as previously described.[1] The field calculated, E_θ, is given by (131), p. 44 of Wait[2] with the slot centered at $\phi_0 = 0$. The function computed is $f(\phi)$ defined by

$$f(\phi) \equiv \left. \frac{E\theta}{\dfrac{V_0 e^{-jkr}}{r}} \right|_{\theta = \pi/2} \equiv \left| f(\phi) \right| e^{j\theta_f}$$

where V_0 = voltage across the center of the slot, $k = 2\pi/\lambda_0$, λ_0 = free space wavelength of wave exciting slot. In the equatorial plane

$$H_\phi = \frac{E\theta}{\sqrt{\mu_0/\epsilon_0}}$$

and all other radiation fields are zero.

Tabulations of the magnitude and phase of $f(\phi)$ for the cases of $1 \leq c \leq 30$ in increments of c of 1 for $1 \leq c \leq 10$ and in increments of c of 2 for $10 \leq c \leq 30$, where $c \equiv ka$, are given in Tables I–XX (pp. 789–792).[3] The angular increments in ϕ were in 5° for $0 \leq \phi \leq 90$ and 2° for $90 \leq \phi \leq 180$. All computations are symmetrical with respect to ϕ. The number of terms taken in the summation, N, of (131) are also given in these tables. It is believed that with this number of terms, five significant figure accuracy is obtained in both the magnitude and phase of $f(\phi)$.

From the radiation patterns corresponding to these tabulations, it is seen that the pattern for a large cylinder approaches that from a plane.[1]

A comparison of these computations with those made by Wait and Kates[4] for the cases of $c = 3$, 5, 8, 12, and 18 reveals that four significant figure agreement exists, as previously noted.[5]

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* Received July 21, 1962.

[1] C. M. Knop and A. R. Battista, "Calculated equatorial plane radiation patterns produced by a circumferential slot on a cylinder," IRE Trans. on Antennas and Propagation, vol. AP-9, pp. 498–499; September, 1961.

[2] J. R. Wait, "Electromagnetic radiation from cylindrical structures," Pergamon Press, New York, N. Y.; 1959.

[3] In these tables the column heading notations are: PHI = ϕ, ABSL VLUE = $|f(\phi)/f(0)|$, PHASE = θ_f, and the floating point decimal system is used, that is, for example, 3.167700E-01 = (3.167700) $\times 10^{-1}$, and 2.96900E 02 = (2.969000) $\times 10^{+2}$.

[4] J. R. Wait and J. Kates, "Radiation Patterns of Circumferential Slots on Moderately Large Conducting Cylinders," Inst. Electrical Engrs. (London), Monograph No. 167R, February, 1956, republished in *Proc. IEE*, vol. 103, pt. c, pp. 289–296; September, 1956.

[5] J. R. Wait and C. M. Knop, "Comments on 'Calculated Equatorial Plane Radiation Patterns Produced by a Circumferential Slot on a Cylinder'," IRE Trans. on Antennas and Propagation, vol. AP-10, p. 211; March, 1962.